Graphing Exponentials- 6.0

Topic: Graphing Growth and Decay Functions **Date:**

Objectives: SWBAT (Graph Exponential Functions)

Main Ideas:	Assignment:

Mother Function Exponential

Parent Function:
$f(x) = b^x, b > 1$

Type of Graph:
Continuous and one-to-one

Increasing or Decreasing:
$\uparrow (-\infty, \infty)$

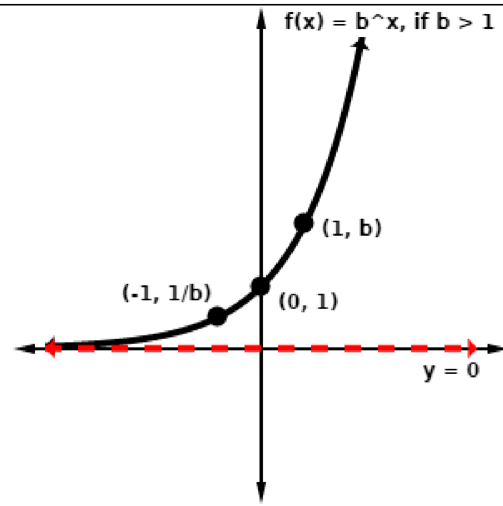
Domain:
$(-\infty, \infty)\{all\ real\ \#'s\}$

Range:
$(0, \infty)$ or $f(x) > 0$

Asymptote:
$x - axis$ or line $(y = 0)$

Intercept(s):
$y - int$ at $(0, 1)$

Max/Min: N/A



NAGS

1. $y = 2^x$

D:

y-int:

R:

x-int:

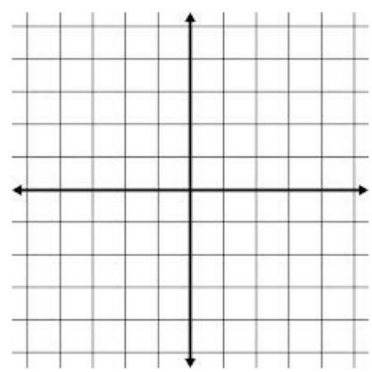
Max/Min:

In/De:

End B:

+/- Intervals:

Asymptote:



2. $y = \left(\frac{1}{2}\right)^x$

D:

y-int:

R:

x-int:

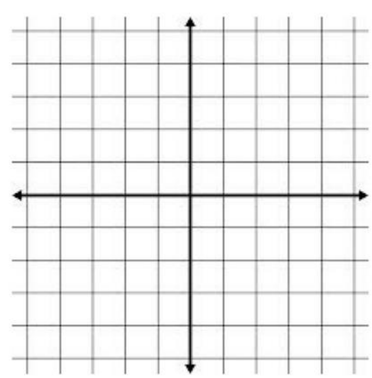
Max/Min:

In/De:

End B:

+/- Intervals:

Asymptote:



Vertex Form

$$f(x) = a \cdot b^{(x-h)} + k$$

<u>$h - value$ ($Horizontal\ Translation$)</u>

- ✓ **h units right if h is positive**
- ✓ **$|h|$ units left if h is negative**

<u>$k - value$ ($Vertical\ Translation$)</u>

- ✓ **k units up if k is positive**
- ✓ **$|k|$ units down if k is negative**

<u>$a - value$ ($Orientation\ and\ Shape$)</u>

- ✓ $If\ a < 0, it\ is\ reflected\ over\ the\ x - axis$
- ✓ $If\ |a| > 1, vertically\ stretch$
- ✓ $If\ 0 < |a| < 1, vertically\ compresseed$

NAGS

3. $y = 3^x - 2$

D:

y-int:

R:

x-int:

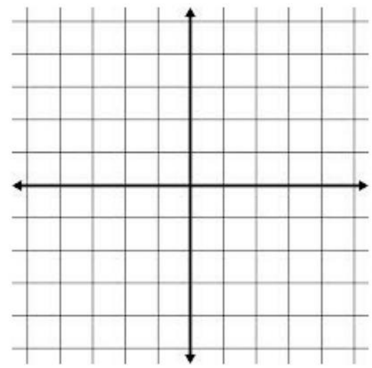
Max/Min:

In/De:

End B:

+/- Intervals:

Asymptote:



4. $y = 2^{x-1}$

D:

y-int:

R:

x-int:

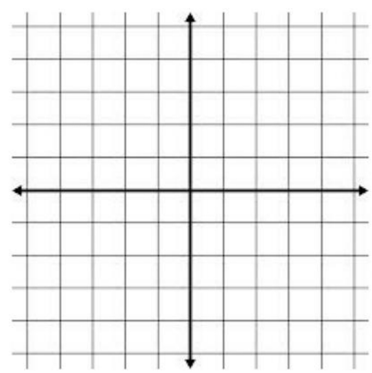
Max/Min:

In/De:

End B:

+/- Intervals:

Asymptote:



Graphing Exponentials- 6.0

Your Turn

5. $y = 2^x - 4$

D:

y-int:

R:

x-int:

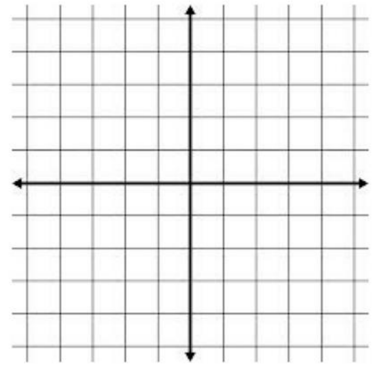
Max/Min:

In/De:

End B:

+/- Intervals:

Asymptote:



6. $y = 2 \cdot 4^{x-2} + 3$

D:

y-int:

R:

x-int:

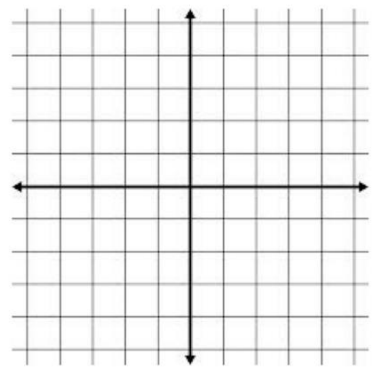
Max/Min:

In/De:

End B:

+/- Intervals:

Asymptote:



7. $y = -3 \cdot \left(\frac{1}{2}\right)^{x-1} + 2$

D:

y-int:

R:

x-int:

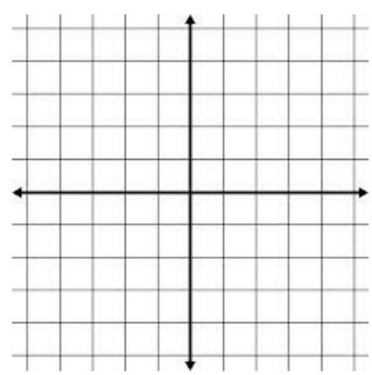
Max/Min:

In/De:

End B:

+/- Intervals:

Asymptote:



Tables of Exponential ($y = a \cdot b^x$)

Exponential Growth

$f(x) = 2 \cdot 3^x$

x	$f(x)$
-2	
-1	
0	
1	
2	
3	

Exponential Decay

$f(x) = 27 \cdot \left(\frac{1}{3}\right)^x$

x	$f(x)$
-2	
-1	
0	
1	
2	
3	

Linear, Quadratic, or Exponential

x	y
-2	-13
-1	-8
0	-3
1	2
2	7

x	y
-2	-3
-1	-6
0	-7
1	-6
2	-3

x	y
-2	16
-1	8
0	4
1	2
2	1

x	y
-2	$-\frac{2}{9}$
-1	$-\frac{2}{3}$
0	-2
1	-6
2	-18

Discovering Euler's number – 6.1

Topic: Compound Interest and Constant *e*	Date:

Objectives: SWBAT (Identify and Discover constant *e* and use Compound Interest)

Main Ideas:	Assignment:
Discovering Euler's Number	Defining "*e*" (Euler's Number)

The number "*e*" is a very famous irrational number, and is one of the most important numbers in mathematics.
-"*e*" is the natural base of logarithms (discovered by John Napier)
-"*e*" is an irrational number (like π)
-The first few digits of "*e*" are;

$2.718281828459045\ldots\ldots$ (never ending, hence irrational)

-"*e*" is a constant <u>not</u> a variable
-"*e*" evolved out of continuous change (or compounded)

Investigation (Let's INVEST!)

Deposited \$100
100% interest
compound interest for 1 year

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

n	$\frac{r}{n}$	$\left(1+\frac{r}{n}\right)$	$\left(1+\frac{r}{n}\right)^{nt}$	$100\left(1+\frac{r}{n}\right)^{nt}$
Yearly				
Quarterly				
Monthly				
Daily				
Hourly				

Compounded Continuously	**Compound Continuously (Need Calculator):** $$A = Pe^{rt}$$
Using "e"	**Find the amount of money after 5 years in an account that started with $1000 and put into an account with an interest rate of 4.5% compounded continuously.**
	A newborn child receives a $20,000 gift toward a college education from her grandparents. How much will the $20,000 be worth in 18 years if it is invested at 7% and compounded continuously?
	A customer invested $2000 in a company at the stock market that earned 2.5% interest compounded continuously. How much would the investment be worth after 5 years?

Logarithms and Their Graphs – 6.2

Topic: Logarithms and Logarithm Functions	Date:

Objectives: SWBAT (Evaluate Logarithm Expressions and Graph Logarithms)

Main Ideas:	Assignment:	
Review	Solve: $4^{2x} = 16^{3x-1}$	Solve: $8^{x-1} = 2^{x+9}$

How would you Solve?

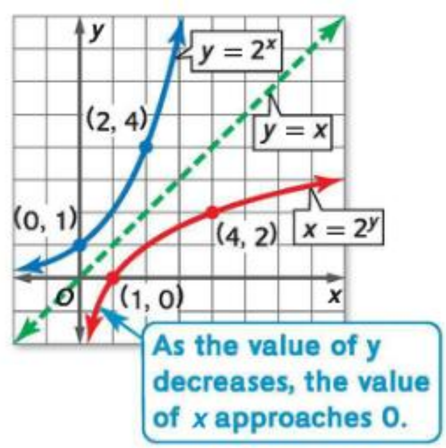
$2x + 7 = 9$ $\quad x^2 - 9 = 0$ $\quad x^3 - 64 = 0$ $\quad 3^x = 9$ $\quad 2^x = 10$

What are Inverses?

What is a LOG?

$y = 2^x$	
x	y
-3	
-2	
-1	
0	
1	
2	
3	

$x = 2^y$	
x	y
$\frac{1}{8}$	
$\frac{1}{4}$	
$\frac{1}{2}$	
1	
2	
4	
3	



Defn of Logarithms

<u>Definition of Logarithms</u>:

Consider,

$$1000 = 10^3$$

Product → 1000; Base → 10; Power or Exponent → 3

Making a general statement of:

$$N = a^x$$

Product → N; Base → a; Power or Exponent → x

If a number N, can be written in the form a^x, then the power (or exponent), x is called the logarithm base a of N.

$$(N > 0,\ and\ a > 0, a \neq 1)$$

$$If\ N = a^x\ then\ log_a N = x$$

Examples:

$$8 = 2^3 \leftrightarrow log_2 8 = 3$$

$$1000 = 10^3 \leftrightarrow log_{10} 1000 = 3$$

$$25 = 5^2 \leftrightarrow log_5 25 = 2$$

How would you say it?

$log_3 9 = 2$ $\qquad$ $log_8 4096 = 4$ $\qquad$ $log_4 64 = 3$ $\qquad$ $log_{10} 10000 = 4$

Practice

Write the following in logarithmic form:

$16 = 2^4$ $\qquad$ $27 = 3^3$ $\qquad$ $\frac{1}{9} = 3^{-2}$ $\qquad$ $x = a^y$

Rewrite the following logarithms as an exponential equation:

$log_5 625 = 4$ $\qquad$ $log_{10} 100000 = 5$ $\qquad$ $log_{10} 0.01 = -2$ $\qquad$ $log_4 \frac{1}{16} = -2$

Logarithms and Their Graphs – 6.2

Evaluating with LOGS

Evaluate

$log_{16}4 = y$

Evaluate

$log_3 81$

Evaluate

$log_3 243 = y$

Evaluate

$log_{10}1000$

Parent Function of Logarithmic Functions

Parent Function: $f(x) = log_b x$

Type of Graph:
Continuous, one-to-one

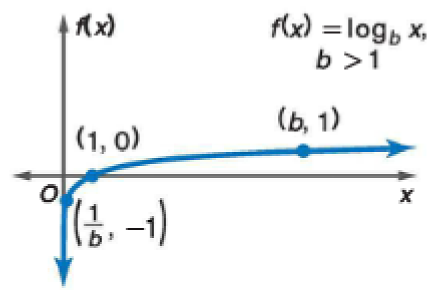
Domain:
$(0, \infty)$ *or* $x > 0$

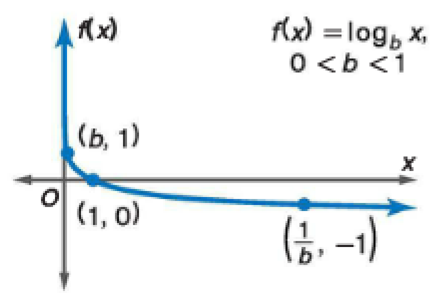
Range:
$(-\infty, \infty)$ *or all real #'s*

Asymptote:
$y - axis\ of\ line\ f(x) = 0$

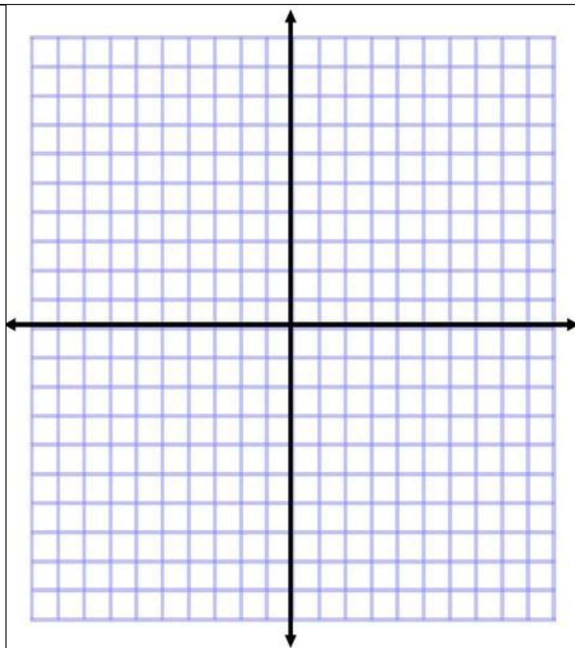
Intercept(s):
$x - int\ at\ (1, 0)$

Max/Min:
N/A





Graphing



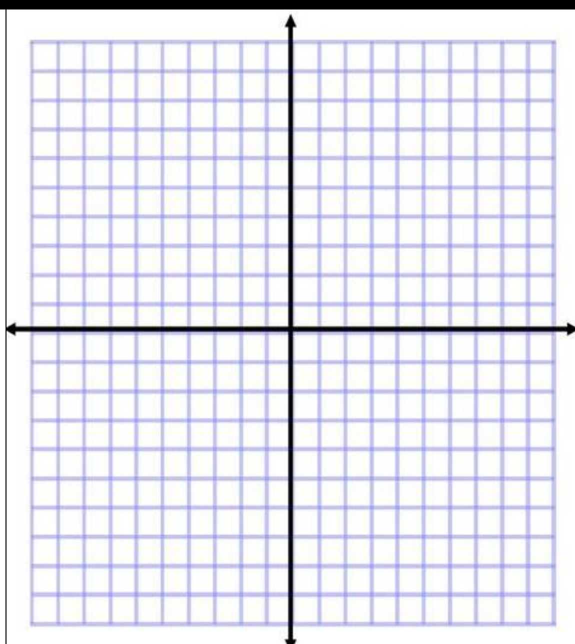
Graph the function $f(x) = log_3 x$.

Step #1: Identify the Base

Step #2: Determine the points on the graph.

Step #3: Plot the points and sketch the graph.

Your Turn



Graph the function $f(x) = log_{\frac{1}{4}} x$.

Step #1: Identify the Base

Step #2: Determine the points on the graph.

Step #3: Plot the points and sketch the graph.

Logarithms and Their Graphs - 6.2

Transformations

$f(x) = a \cdot log_b(x-h) + k$

<u>$h - value$ (*Horizontal Translation*)</u>

- ✓ h units right if h is positive
- ✓ $|h|$ units left if h is negative

<u>$k - value$ (*Vertical Translation*)</u>

- ✓ k units up if k is positive
- ✓ $|k|$ units down if k is negative

<u>$a - value$ (*Orientation and Shape*)</u>

- ✓ *If* $a < 0$, *it is reflected over the* $x - axis$
- ✓ *If* $|a| > 1$, *vertically stretch*
- ✓ *If* $0 < |a| < 1$, *vertically compresseed*

$f(x) = 4log_2(x-7) + 5$

Parent is $f(x) = log_2 x$

$a =$ $\qquad$ $b =$ $\qquad$ $c =$ $\qquad$ $d =$

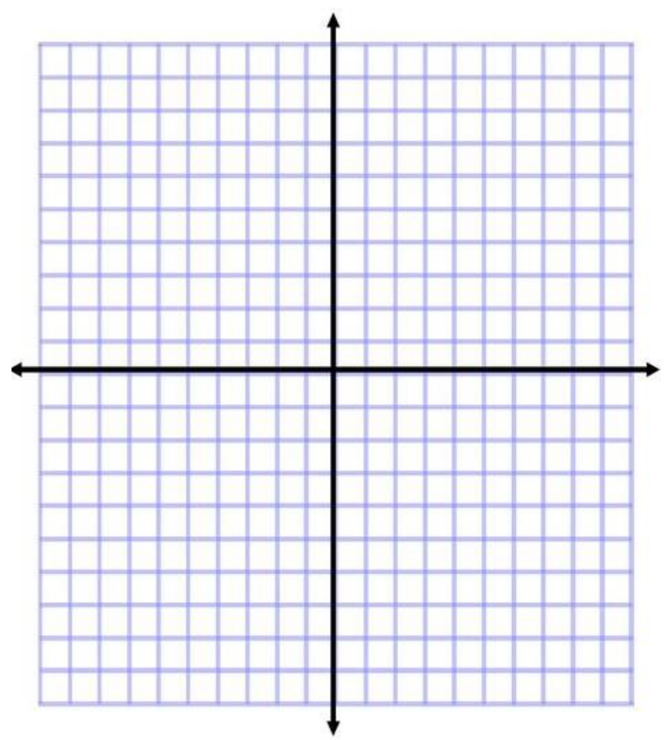
Graph

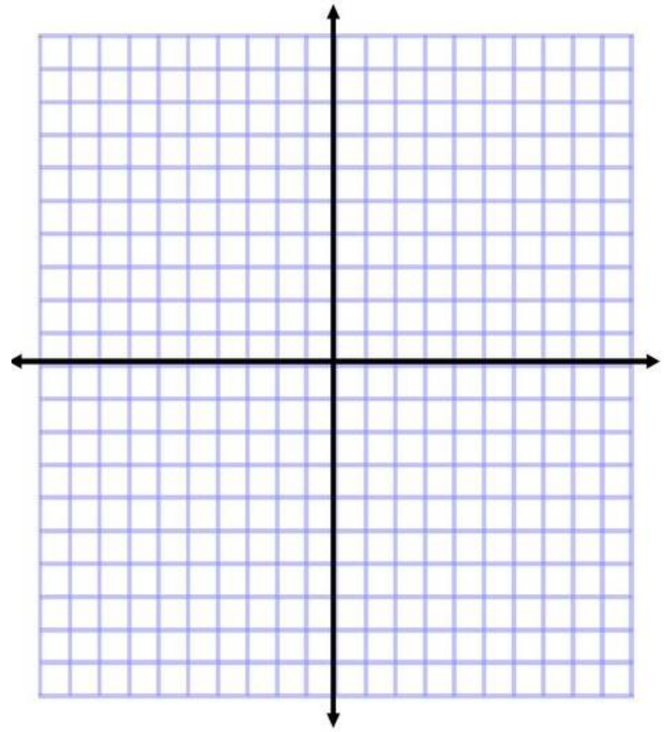
Graph the function $f(x) = \frac{1}{3}log_6 x - 1$

Identify the parent:

Identify parts:

Sketch the Graph



Your Turn	**Graph the function $f(x) = 4log_{\frac{1}{3}}(x+2)$** **Identify the parent:** **Identify parts:** **Sketch the Graph** 

Solving Logarithmic Equations – 6.3

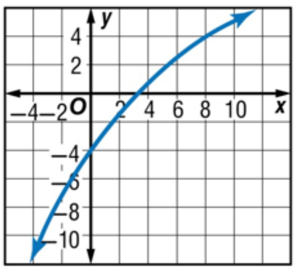
Topic: Solving Logarithmic Equations	Date:

Objectives: SWBAT (Solve Logarithmic Equations using the corresponding Exponential EQ.)

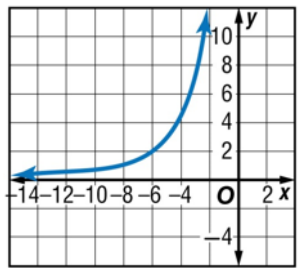
<table>
<tr><th>Main Ideas:</th><th colspan="2">Assignment:</th></tr>
<tr><td rowspan="2">Review</td><td>Write $log_6 216 = 3$ in exponential form.</td><td>Write $4^{-3} = \frac{1}{64}$ in logarithmic form.</td></tr>
<tr><td colspan="2">

Choose the correct graph for $f(x) = 2log_2 x$.

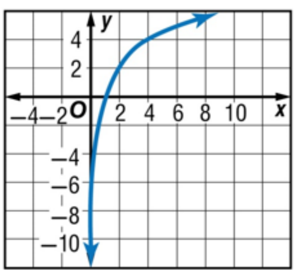
A.



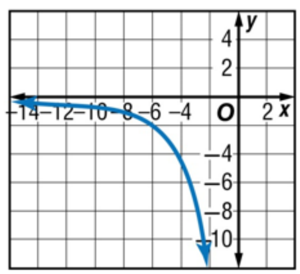
C.



B.



D.



</td></tr>
<tr><td rowspan="3">Solving LOGS</td><td colspan="2">We can use the Definition of logarithms to solve logarithmic equations. Using the corresponding exponential equation that goes with it.</td></tr>
<tr><td>Solve.
$log_{36} x = \frac{3}{2}$</td><td>Solve.
$log_9 x = \frac{3}{2}$</td></tr>
<tr><td>Solve.
$log_{16} x = \frac{5}{2}$</td><td>Solve.
$log_8 x = \frac{4}{3}$</td></tr>
</table>

Property of Equality

Property of Equality for Logarithmic Functions

Symbols:

If b is a positive number other than 1, then $log_b x = log_b y$ if and only if $x = y$.

Example:

If $log_5 x = log_5 8$, then $x = 8$. If $x = 8$, then $log_5 x = log_5 8$.

Solve:

$$log_2(x^2 - 4) = log_2 3x$$

Your Turn

Solve.

$$log_4 x^2 = log_4(-6x - 8)$$

Solve.

$$log_4 x^2 = log_4(x + 20)$$

Solve.

$$log_3(x^2 - 15) = log_3 2x$$

Solve.

$$log_7(x^2 - 4) = log_7(-x + 2)$$

??

Find $log_3 27 + log_9 27 + log_{27} 27 + log_{81} 27 + log_{243} 27$

Properties of Logarithms – 6.4

Topic: Properties of Logarithms	Date:

Objectives: SWBAT (Simplify and evaluate expressions using properties of LOGs)

Main Ideas:	Assignment:

LAWS OF LOGS

Glue Foldable Here

Using Laws of LOGS

Using the laws for log of a product and log of a quotient to simplify:
(to condense to 1 logarithm)

$log5 + log2$	$log_6 7 + log_6 4 - log_6 2$
$log_2 2 + log_2 3 - log_2 7 - log_2 6$	$log3 + log4 + log\left(\frac{1}{2}\right) + log\left(\frac{1}{6}\right)$

All Laws

Using the law of logarithms, simplify (condense) and evaluate:

$log_2 15 + log_2 14 - log_2 105$	$log_3 9 - log_3\left(\frac{1}{3}\right)$
$log_a 1 + log_a 1 + log_a 1$	$log_2\left(\frac{1}{8}\right) - log_2\left(\frac{1}{64}\right)$

$$log_2 2^8 + log_2\left(\frac{1}{8}\right)^2$$

Properties of Logarithms - 6.4

Tough

Use law of logs to rewrite as a single logarithm. (Condense)

$3(log_5 3 + log_5 x - log_5 y)$	$8(log_7 x + 5log_7 y) - 2log_7 z$

Tougher

Use law of logs to rewrite as separate logarithms. (Expand)

$log_2\left(\frac{x^2 z}{y^3}\right)$	$log\left(\frac{y^4}{xz^5}\right)^2$

Evaluating Logs

Let $log_m B = 4$ and $log_m C = 6$. Calculate each value.

$log_m\left(\frac{1}{C}\right)$	$log_m B^3 C^2$

$$log_m\left(\frac{B^2}{C}\right)$$

LAWS of LOGS Cont. – 6.5

Topic: Day 2 Of LAWS of LOGS	Date:

Objectives: SWBAT (Use Properties of Logarithms to Evaluate and Solve)

Main Ideas:	Assignment:	
Mixed Review	Solve. $log_4(x^2 - 30) = log_4 x$	Solve. $log_5(x^2 - 2x) = log_5(-5x + 10)$
	Condense. $log_3 x + log_3 y - 2 \cdot log_3 z$	Expand. $log_7\left(\frac{x}{y^3 z}\right)^2$
Using LAWS	Use $log_5 2 \approx 0.4307$ to approximate the value of $log_5 250$.	
	Given $log_2 3 \approx 1.5850$, what is the approximate value of $log_2 96$?	
	Use $log_4 3 \approx 0.7925$ to approximate the value of $log_4 192$.	

More LOGS	Given that $log_5 6 \approx 1.1133$, approximate the value of $log_5 216$.	
	Given that $log_4 6 \approx 1.2925$, what is the approximate value of $log_4 1296$?	
Using LAWS to Solve	Solve. $4log_2 x - log_2 5 = log_2 125$	Solve. $2log_3(x-2) - log_3 6 = log_3 150$
	Solve. $log_6 x + log_6(x-9) = 2$	Solve. $2log_7 x = log_7 27 + log_7 3$

Common Logarithms – 6.6

Topic: Common Logarithmic | **Date:**

Objectives: SWBAT (Solve Exponential Equations using Common Logarithms)

Main Ideas: | **Assignment:**

Common LOG

There's a special exponential that we haven't talked about yet:

$$y = 10^x$$

He has a special inverse as well:

$$y = log_{10}x$$

This *log* turns up a lot (chemistry, biology, geology, sound engineering, and so on…), so we call it "*the common log*". It even has its own button on the calculator!

LOG

See it? Notice they leave the base, 10, off?

It's because this is the most commonly used *log*, so the 10 is just assumed. From now on when you see…

$$y = logx\text{, it's really } y = log_{10}x$$

Just like the 2 as the index of square root.

$$\sqrt[2]{x} = \sqrt{x}$$

Approximating

Finding Common Logarithms

Use a calculator to evaluate $log6$ to the nearest ten-thousandth.	**Use a calculator to evaluate $log0.35$ to the nearest ten-thousandth.**
Find the approximate value of $log5$ to the nearest hundredth.	**Find the approximate value of $log0.62$ to the nearest hundredth.**

Using Common LOG to Solve	**Sometimes you cannot use "Same Base Method" to solve exponentials.....so you will use the common logarithm and LAWS of LOGS to solve.** **Solve. Round to the nearest ten-thousandth.** $4^x = 19$	
Your Turn	**Solve.** $3^x = 15$	**Solve.** $6^x = 42$
	Solve. $2^x = 27$	**Solve.** $35 = 4^x$
Change of Base Formula	**Symbols: For all positive numbers *a*, *b*, and *n*, where *a* ≠ 1.** $log_a n = \frac{log n}{log a}$ $\leftarrow$ *common log of original number* / $\leftarrow$ *common log of old base* **Example:** $log_3 11 = \frac{log_{10} 11}{log_{10} 3}$	
	Express $log_5 140$ in terms of common logarithms. Then round to the nearest ten-thousandth.	**What is $log_5 16$ expressed in terms of common logarithms?**

Base "e" and Natural LOG – 6.7

Topic: Base "e" and Natural LOG	Date:

Objectives: SWBAT (Evaluate Expressions and Solve Equations with Base e and LN)

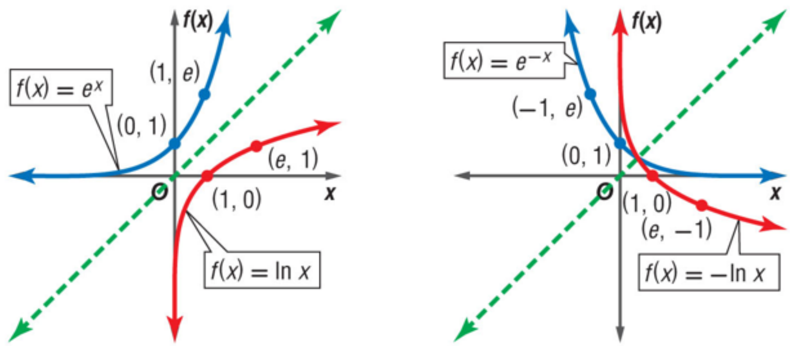
Main Ideas: | **Assignment:**

Natural Base

The function $f(x) = e^x$ is used to model continuous exponential growth.

The function $f(x) = e^{-x}$ is used to model continuous exponential decay.

The inverse of a natural base exponential function is called the natural logarithm. This logarithm can be written as $log_e x$, but is more often abbreviated as lnx.



Exponential Growth | Exponential Decay

Multiple Representations

Write an equivalent logarithmic equation for $e^x = 23$.	Write an equivalent logarithmic equation for $e^4 = x$.
What is $e^x = 15$ in logarithmic form?	What is $e^4 = x$ in logarithmic form?
Write $lnx \approx 1.2528$ in exponential form.	Write $ln25 \approx x$ in exponential form.
Write $lnx \approx 1.5763$ in exponential form.	Write $ln47 = x$ in exponential form.

Condensing	Write $4 \cdot ln3 + ln6$ as a single logarithm.	Write $2 \cdot ln3 + ln4 + lny$ as a single logarithm.
	Write $4 \cdot ln2 + In3$ as a single logarithm.	Write $3 \cdot ln3 + ln\frac{1}{3} + lnx$ as a single logarithm.
Solving Equation with Natural Base “e” and LOG	Solve $3e^{-2x} + 4 = 10$. Round to the nearest ten-thousandth.	What is the solution to the equation $2e^{-2x} + 5 = 15$?
	Solve $2 \cdot ln5x = 6$. Round to the nearest ten-thousandth.	Solve the equation $3 \cdot ln6x = 12$. Round to the nearest ten-thousandth.

Base "e" and Natural LOG – 6.7

Compounded Continuously

Continuously Compounded Interested

$$A = Pe^{rt}$$

$A = Ending\ Amount\ of\ Money$
$t = amount\ of\ time\ in\ years\ in\ account$
$r = annual\ percentage\ rate$
$P = Principal\ or\ Beginning\ amount\ invested$

Suppose you deposit $700 into an account paying 3% annual interest, compounded continuously. What is the balance after 8 years?

Suppose you deposit $700 into an account paying 3% annual interest, compounded continuously. How long will it take for the balance in your account to reach at least $1200?

Suppose you deposit $700 into an account paying 3% annual interest, compounded continuously. How much would have to be deposited in order to reach a balance of $1500 after 12 years?

Suppose you deposit $700 into an account paying 6% annual interest, compounded continuously. How long will it take for the balance in your account to reach at least $2500?

Using Expo and LOG Functions – 6.8

Topic: Using Exponential and Logarithmic Functions	**Date:**

Objectives: SWBAT (Use logarithms to solve problems with expo growth and decay.)

<table>
<tr><td>Main Ideas:</td><td colspan="2">Assignment:</td></tr>
<tr><td rowspan="2">Review</td><td>Solve $6 + 4e^{-x} = 12$. Round to the nearest ten-thousandth.</td><td>Write an equivalent logarithmic function for $e^6 = y$.</td></tr>
<tr><td>Write $6 \cdot lnx - lnx^2$ as a single logarithm.</td><td>You deposit $2000 in an account paying 4% annual interest compounded continuously. Using the formula $A = Pe^{rt}$, how long will it take for your money to double?</td></tr>
<tr><td>Continuous Growth/Deca</td><td><u>Continuous Exponential Growth</u>
$$f(x) = ae^{kt}$$
$a = initial\ value$
$t = time\ in\ years$
$k = constant\ rate\ of\ continuous\ growth$</td><td><u>Continuous Exponential Decay</u>
$$f(x) = ae^{-kt}$$
$a = initial\ value$
$t = time\ in\ years$
$k = constant\ rate\ of\ continuous\ growth$</td></tr>
<tr><td>Examples</td><td colspan="2">The half-life of Sodium-22 is 2.6 years. Determine the value of k and the equation of decay for Sodium-22.</td></tr>
</table>

More Examples

The half-life of radioactive iodine used in medical studies is 8 hours. What is the value of *k* for radioactive iodine?

A geologist examining a meteorite estimates that it contains only about 10% as much Sodium-22 as it would have contained when it reached the surface of the Earth. How long ago did the meteorite reach the surface of the Earth?

The half-life of radioactive iodine used in medical studies is 8 hours. A doctor wants to know when the amount of radioactive iodine in a patient's body is 20% of the original amount. When will this occur?

In 2007, the population of China was 1.32 billion. In 2000, it was 1.26 billion. Determine the value of *k*, China's relative rate of growth.

In 2007, the population of China was 1.32 billion. In 2000, it was 1.26 billion. When will China's population reach 1.5 billion?